$$T=t_1+t_2=\sqrt{\frac{2x}{g}}+\sqrt{\frac{2}{g}}[e\sqrt{x}\pm\sqrt{(e^2x-a)}].$$

Putting $dT/dx=0$, we get $x=\dfrac{a(1+e)^2}{e^2(1+2e)}$.

Also solved by C. N. Schmall, and H. Prime. In their solutions, these gentlemen use $e=1$.

**324. Proposed by V. M. SPUNAR, Chicago, Ill.**

A parabola slides between two rectangular axes; find (a) the locus of the focus, and (b) the locus of the vertex.

**I. Solution by ELIJAH SWIFT, Princeton University.**

Take the parabola in the form $y^2=4ax$. The equation of any two perperdicular tangents may be written $y=mx+(a/m)$ and $y=-(1/m)x-am$. The distances of these from the focus are $Y=(a/m)\sqrt{(1+m^2)}$ and $X=a\sqrt{(1+m^2)}$, and these quantities would therefore be the coördinates of the focus of a moving parabola, if the two perpendicular tangents were axes. Eliminating $m$, we get as the locus

$$x^2y^2=a^2(x^2+y^2),\text{ or in polar coördinates, } r=\frac{2a}{\sin 2\phi}.$$

In case (b), $Y=\dfrac{a}{m\sqrt{(1+m^2)}}$, $X=\dfrac{am^2}{\sqrt{(1+m^2)}}$.
Elimination of $m$ gives us $x^2y^2(x^{\frac{2}{3}}+y^{\frac{2}{3}})^3=a^6$.

**261. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.**

A man six feet high, walking at a rate of 100 yards a minute, crosses a muddy road close behind a wheel of a carriage which is going thrice as fast and in a direction at right angles to that of the man's motion. The diameter of the wheel is five feet. If, when the man is four feet from the middle of the wheel the mud is splashed up to the hight of seven feet, will any of it touch him? Unsolved in *Educational Times*.

**Solution by H. PRIME, Boston, Massachusetts.**

Let $\phi$ be the angle of elevation, $v$ the velocity, of particles of mud leaving the ground at $A$; let the line $ABH$ be the path of the wheel, $DEFGHI$ the man's path, the man being at $D$ when the wheel touches $A$, at $F$ when it touches $B$, and the mud is 7 feet high. Then $BF=4$ feet, $BH=3FH-\frac{5}{2}$, $16-FH^2=(3FH-\frac{5}{2})^2$, $FH=1.989960$, $BH=3.469879$.

The man is at three critical points in his path,—at $E$ when the mud is 6 feet high rising, at $G$ when it is 6 feet high falling, at $I$ when the mud is striking the ground. While the man is between $E$ and $G$, no mud can touch him, for it is more than 6 feet high. Between $D$ and $E$ only rising mud can hit him, between $G$ and $I$ only falling mud.

If $g=32$, the mud rises 7 feet, or falls 7 feet, in $\sqrt{\frac{7}{16}}$ seconds, and rises the last foot, or falls the first foot, in $\frac{1}{4}$ second. While the mud is rising, the carriage goes from $A$ to $B$, 15 feet a second. Hence $AB=15\sqrt{\frac{7}{16}}=9.921568$, $AH=13.391447$; also $EF=FG=1.25$, $DF=FI=3.307189$; $EH=3.239560$, $GH=0.739560$, $HI=1.317630$; hence $AE=13.778007$, $AG=13.412149$, $AI=13.456409$.

For determining the limiting values of $\phi$ or $v$ we have the equation of the parabola in which a projected particle of mud moves, $A$ being the origin, $y=x\tan\phi-gx^2/2v^2\cos^2\phi$; also the maximum height$=v^2\sin^2\phi/2g=7$. Whence $\tan\phi=[14\pm\sqrt{(196-28y)}]/x$.
At $E$, $x=AE$, $y=6$. Hence $\phi=32°\ 17'\ 43''$ or $v=39.6158$ feet a second.
At $G$, $x=AG$, $y=6$. Hence $\phi=55°\ 11'\ 30''$ or $v=25.7787$ feet a second.
At $I$, $x=AI$, $y=0$. Hence $\phi=64°\ 19'\ 54''$ or $v=23.5149$ feet a second.

Thus, if $32°\ 17'\ 43''<\phi<55°\ 11'\ 30''$, or if $\phi>64°\ 19'\ 54''$, none of the mud touches the man. For in the former case it passes over his head, in the latter it does not reach to him. The man is hit between $D$ and $E$ if $\phi<32°\ 17'\ 43''$, or between $G$ and $I$ if $55°\ 11'\ 30''<\phi<64°\ 19'\ 54''$.

Also solved by the Proposer.

**262. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.**

A hemispherical shell, whose radius is equal to the mean radius of the earth and whose thickness is one centimeter, is constructed of a matter whose density is equal to the mean density of the earth. A particle starts from rest at the center of the shell under the action of the attraction of the shell. Express as the decimal of a year the time it takes the particle to reach the surface of the shell, and find the velocity in centimeters per second of the particle just before it reaches the shell. Unsolved in *Educational Times*.

**I. Solution by the PROPOSER.**

$$\frac{\text{The total volume of the earth}}{\text{The volume of the shell}}=\tfrac{2}{3}R, \text{ nearly}; R=\frac{40,000,000}{2\pi}m=$$

$\frac{2}{\pi}10^9 cm$, the mean radius of the earth.

$$\therefore \frac{\text{The mass of the earth, } M'}{\text{The mass of the shell, } M}=\frac{147\times10^9 R^2}{M}=\tfrac{2}{3}R, \text{ at the same mean}$$
density of the earth.

$$\therefore M=\tfrac{3}{2}\times147\times10^9 Rkg \ (R \text{ in meters})=\frac{441\times10^{19}}{\pi}g.$$

The center of gravity, $O$, of the mass $M$ is the center of gravity of the circular arc on the axis of symmetry ($X$-axis) at the distance $(2/\pi)R$ from $O'$, the center of the shell.

The force of attraction, $F$, exerted by $O$ on the particle (mass=1) in any position $P$ between $O'O$ is $CM/x^2$, where $x$=abscissa of $P$ and $C$, the constant of gravitation$=6.5\times10^{-8}$ (Baily). $C$ becomes, however, 1 if cm.—sec. would be chosen and $1.537\times10^7 g$ as unity of the mass, $M$. Hence, $MC=\frac{441\times10^{15}}{1537\pi}$. Then,